# Coursework guide

## AQA GCSE Coursework Guide

The following guide applies to students taking AQA GCSE Linear (A) and Modular (B) specifications (4301 and 4302) and is written to advise teachers. The specifications are available on the AQA website (www.aqa.org.uk).

### What is required for AQA GCSE coursework?

Each pupil is required to produce two pieces of coursework. The first, AO1, tends to be algebraic or spatial in nature. The second, AO4, is a statistical piece.

Candidates are entered for either Option T, for which the coursework is marked by the centre, or Option X, for which it is marked by AQA. Option X candidates must select from the AQA-set tasks listed below, whereas Option T candidates may select from the AQA-set task list or work on a task set by the centre.

The six AQA-set tasks for AO1 are:

- Spacers
- Trios
- Fraction Differences
- Equable Shapes
- Number Grid
- Trays (http://www.aqa.org.uk/qual/pdf/AQA-3301-3302-AO1-Set-Tasks.pdf)

and for AO4 are:

- Reaction Times
- Guestimate
- Where in the World?
- Read All About It
- Pulse Rate
- Memory Game (http://www.aqa.org.uk/qual/pdf/AQA-3301-3302-AO4-Set-Tasks.pdf).

An AO1 task is not expected to take longer than a fortnight (approximately three hours of official writing time plus some additional time to work on the problem in rough). An AO4 task by its very nature will take longer. Some centres have been known to spend half a term on a single piece of coursework. Alternatively, some centres have chosen to use one lesson each week or fortnight through the school year.

All tasks are marked according to a three-strand, eight-level system. AO1 tasks are assessed by Making and Monitoring Decisions, Communicating Mathematically and Mathematical Reasoning, AO4 tasks by Specify and Plan, Collect, Process and Represent Data, and Interpret and Discuss.

## Getting the best marks from your students

Both AO1 and AO4 tasks contain particular components that many students find difficult. AO4 tasks are generally found more complicated so these are discussed first. The main components that students need advice on for AO4 tasks are constructing hypotheses and interpreting their graphs. For AO1 tasks, students are usually able to collect results, but then find difficulty when it comes to generalising their results and proving their formulae.

## AO4 Tasks

The point of an AO4 task is to investigate a hypothesis using appropriate statistical techniques. The word *appropriate* is important – too little analysis or extra redundant analysis are both penalised.

The AQA set tasks simply provide a scenario for which students need to develop an hypothesis, collect meaningful data and analyse it by means of statistical diagrams. Set tasks for other examination boards often have data which has already been collected and just needs to be sampled (e.g. Mayfield High School, data for which is available on the Edexcel website http://www.edexcel.org.uk/VirtualContent/97299/006_Mayfield_High_School_Datasheet.xls). Using data which has already been collected is allowed, provided that is used appropriately. Experience suggests that using data which has already been collected saves a lot of time which could be used far more productively.

### *Constructing hypotheses*

Students are often able to suggest the basic hypotheses for marks 1 to 6, but often struggle when trying to produce a hypothesis which requires 'creative thinking' for marks 7 and 8. They should appreciate that it does not matter whether or not their hypothesis turns out to be correct, incorrect or undecided, but that their conclusions should be valid.

*There is no relationship between people's reaction times in different situations.*

This hypothesis is rather limiting. To some degree it can be proved or disproved by conducting two experiments on 50 or so people and plotting a scatter diagram, but it can only lead to higher marks if the hypothesis is now refined.

*Boys are quicker than girls.*

This hypothesis is better than the one above because it immediately enables the student to compare two groups of people and use stratified sampling. However, it is not a particularly inspired task and could easily be improved to incorporate additional factors.

*Girls react more quickly than boys as they get older.*

This hypothesis not only enables the student to compare genders, but ages as well. The student could, for example, look at the differences between the genders in years 7, 9 and 11 and how they change.

*Girls react more quickly than boys in emotional situations, whereas boys react more quickly than girls in physical situations.*

This hypothesis is based upon stereotypes, but is a good example of creative thinking. It is also a task that the student will probably be interested in tackling and ownership of a problem is very important in an AO4 investigation. This sort of hypothesis also lends itself to meaningful interpretation; for example, justifying or dispelling stereotypes, thinking about gender differences in sport, money in sport, and human relationships.

| *Marks* | *What is expected* |
|---|---|
| 2 | The problem is simple and well-defined. |
| 4 | The problem is simple and involves the routine use of simple statistical techniques, but the aims are clear and the plan is described fairly well. |
| 6 | The problem is more complex. The aims are stated in statistical terms and the plan is appropriate. |
| 8 | The problem requires creative thinking and careful specification. The aims are stated in statistical terms and the plan is appropriate with reasons for choices. |

### *Interpreting graphs*

For AO4 tasks, the strand that students tend to find most demanding is the third, 'Interpret and Discuss'. The following examples demonstrate the difference between good and bad interpretation. It is not difficult to work out which is which.

*My histogram is a weird shape and does not tell me anything.*

Although this student might appreciate common shapes of distributions such as the Normal, this is unclear. No matter what shape the histogram, it can still be interpreted (it will "tell me something").

*My histogram suggests that reaction time is not distributed symmetrically. Although it is bell-shaped to some degree, it is not sufficiently bell-shaped to be described as Normal. This means that I will not use the mean and standard deviation to compare my data; instead, I will use box plots.*

This student appreciates that although her histogram is not an expected shape, it *can* be used to inform her next decision.

*Box plot A has a median of 0.36. Box plot B has a median of 0.32. A is larger on average than B.*

This student can read and compare her box plots, but only superficially. There is no attempt to interpret, not even an attempt to explain to what the figures are referring.

*Looking at the two box plots, the girls have an average reaction time of 0.36 s whereas the boys have an average reaction time of 0.32 s. This suggests that boys react more quickly than girls on average in this type of situation (catching a falling ruler). There might, however, be different situations for which boys do not react more quickly. Comparing the averages, girls take 12.5% longer to react in this experiment. This is a significant difference and suggests that my conclusion is correct.*

This student has also compared and interpreted her box plots, but she has done this in context. She has also considered a limitation of her comparison. By comparing the relative differences between the medians this student is able to determine how significant her results are.

| ***Marks*** | ***What is expected*** |
|---|---|
| 2 | Students can comment on patterns, but make little attempt to relate the results to their hypothesis. |
| 4 | Students can comment on patterns and attempt to relate results to their hypothesis, even if some of their conclusions are incorrect. |
| 6 | Students comment on patterns and suggest reasons for exceptions. Their conclusions are correct and their inferences are appropriate. They understand statistical significance. They evaluate the effectiveness of the strategy and make a simple assessment of limitations. |
| 8 | Students comment on patterns and suggest plausible reasons for exceptions. Their conclusions are correct, their inferences are detailed and they use the language of probability. They understand statistical significance. They evaluate the effectiveness of the strategy and recognise limitations, making suggestions for improvement. |

In general:

- Students should be aware that planning, diagrams and interpretation are equally weighted and therefore must be given equal importance in their scripts. Five lines of planning and interpreting separated by fifty pages of diagrams is not appropriate!
- When interpreting, students should not just be commenting that such and such is bigger than this or has a larger range. They should make comparisons in context. They should be 'thinking outside the box' when it comes to making observations.
- Students should make individual points relating to each of their smaller hypotheses, then interpret everything as whole to discuss their demanding hypotheses.
- Students should be aware that the difference between, say, a mean of 56.3 kg and 56.6 kg is negligible and therefore the fact that one mass is higher than the other is insignificant. A typical significant difference would be about 5%. Hence students should check the percentage difference between their results when comparing medians, interquartile ranges, etc.
- A helpful (and expected) way of interpreting results is through the language of probability. For example, 'The probability of a boy catching a falling ruler faster than the median time for both genders is 0.62, whereas the probability of a boy detecting a hazard in a driving test faster than the median time for both genders is 0.45. Hence a boy is more likely to be better than a girl at catching a falling ruler quickly. Maybe women are better drivers than men, but men are better at sports such as cricket.'

### *Conclusions*

Students are expected to conclude their findings, but should be going further than just summarising. Firstly, they should be drawing all their little observations together to produce an overall decision about their demanding hypothesis. Other points that students should also discuss include:

- Did anything go wrong during the task? How did the student deal with problems?
- How did the student try to eliminate bias (e.g. did the size of the sample affect the validity of the conclusions?)
- What might the student do to look at the hypothesis as a country-wide phenomenon?

- Was the student surprised at what she discovered? Can she think of any explanations as to why what she found should be the case?

## AO1 Tasks

The point of an AO1 task is to investigate an aspect of pure mathematics, starting by looking at specific examples before generalising results usually into a formula which is to be proved. AO1 tasks can be either algebraic (e.g. *Trios*) or spatial (e.g. *Equable Shapes*).

An AO1 task is usually introduced with an example (but not the simplest). For example, *Spacers* starts with a wall which is three squares on each side. The usual way to start an AO1 task is to 'start small' (i.e. a wall which is one square) and then look at more examples systematically, recording results and tabulating them with a view to finding a pattern and then a rule.

Some students may be unable to work on paper initially and need to use models, such as making trays for the *Trays* task or using Cuisinere rods for *Trios*.

A good script will demonstrate a logical process, but still be concise. For high marks, algebra should be introduced as soon as possible. Students tend to find the proof stage the hardest of all. Although it is often possible to stumble upon a result, it is not then clear how to prove it in general. Theoretically proofs should be possible using mathematics they should have studied for GCSE, but often they will need to research further. Given below are two examples of proofs, one which is geometric and one which uses mathematics from beyond the GCSE syllabus.

### *Geometric proof – Trios*

*Please note that the following discourse is only meant to illustrate how a geometric proof might be employed. It is not expected that a teacher should describe this proof to a pupil working on the Trios task.*

The *Trios* task produces triangle numbers:

| Total | Number of ways |
|---|---|
| 3 (e.g. (1, 1, 1) | 1 |
| 4 (e.g. (1, 1, 2) | 3 |
| 5 (e.g. (2, 2, 1) | 6 |
| 6 (e.g. (3, 1, 2) | 10 |
| 7 (e.g. (1, 5, 1) | 15 |

The formula, using N for number of ways and T for total is given by $N = \frac{1}{2}(T - 1)(T - 2)$.

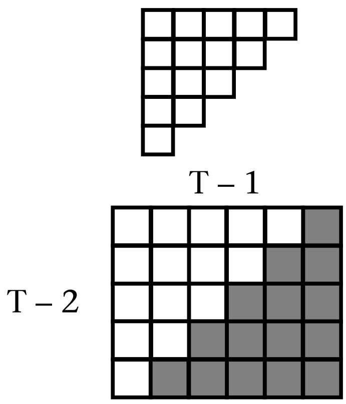
Probably the best way to show how this formula comes about is by representing the possible trios for a certain trio in a triangular pattern to highlight why triangle numbers occur.

For example, using a total of 7:

| | | | | |
|---|---|---|---|---|
| (1, 1, 5) | (1, 2, 4) | (1, 3, 3) | (1, 4, 2) | (1, 5, 1) |
| (2, 1, 4) | (2, 2, 3) | (2, 3, 2) | (2, 4, 1) | |
| (3, 1, 3) | (3, 2, 2) | (3, 3, 1) | | |
| (4, 1, 2) | (4, 2, 1) | | | |
| (5, 1, 1) | | | | |

The top row contains all the trios which begin with a 1, the second row all those that begin with 2, etc. The second and third digits of each trio are systematically listed in ascending order (e.g. 1, 5   2, 4   3, 3, etc).

Hence each total can have its trios arranged in a triangular pattern with the trios in each row beginning from 1 to (T – 2) for the (T – 2, 1, 1) trio.




By rotating an identical pattern, it is possible to make a rectangle with sides of (T – 1) and (T – 2) of which the original pattern makes up half. Hence there are $\frac{1}{2}(T - 1)(T - 2)$ possible trios for each total.

## *Proofs beyond GCSE specification – Trays*

For the *Trays* task, it is fairly straightforward to determine that the areas will be equal when $x = 3$. However, finding a proof for the value of $x$ which makes the volume a maximum is far more difficult. A spreadsheet can be used to show that $x = 3$ gives the largest volume compared to other discrete values and a graph will make this even clearer for continuous values.

Differentiation will clearly find the value of $x$ which makes the volume a maximum, but since it is not on the GCSE specification, students will have to perform research to discover the technique. It is not unprofessional to direct students to appropriate sources (e.g. internet websites, A-level texts) or to teach differentiation on request. The same, of course, is true for other topics which are beyond the specification. Students must, however, demonstrate that they have an understanding of the new technique.

| *Mark* | *What is expected* |
|---|---|
| 1 | Students are able to illustrate other examples of the original problem. |
| 2 | Students are investigating additional examples of the original problem and looking for a pattern. |
| 3 | Students can make their own general statements and explain their reasoning. |
| 4 | Students have made generalisations which they are beginning to justify and they can check their generalisations using new examples. |
| 5 | Students justify generalisations. |
| 6 | Students examine generalisations. |
| 7 | Justifications must now include a number of features/variables. |
| 8 | Justification must be mathematically rigorous and students must consider the conditions under which the justification remains valid. |

## Suggestions for other tasks

Should students take Option T, it is not compulsory to use any of the AQA set-tasks. Listed below is a selection of some alternatives, most of which are used or have been used by other examination boards.

### *AO1 Tasks*

***Fencing Problem*** A farmer has 1000 metres of fencing and wishes to enclose the maximum possible area. Students have to investigate which shape has the maximum area for a given perimeter. This investigation makes good use of a spreadsheet when investigating the areas of the regular polygons. Suitable for both tiers.

***Emma's Dilemma*** Students begin by finding the number of arrangements there are in the name EMMA (there are 12). They then investigate the number of arrangements for words of different lengths and different repetitions. Their aim is to find a formula for a word of any length and any number of repeated letters. This investigation is a good introduction to combinations and permutations. Suitable for Higher.

***Pay Phone Problem***

A payphone only accepts 10p and/or 20p coins (or 10p and/or 50p coins) where 10p, 20p, 10p is considered different to 20p, 10p, 10p. Students determine the number of ways of paying for phone calls of different prices. Their aim is to find a solution for the number of ways in a general case with two coins. This investigation is essentially a Fibonacci task. Suitable for both tiers.

***Consecutive Numbers***

Students look at the relationship between the number of consecutive numbers one adds together and their sum. Possible extensions include determining numbers which cannot be written as a sum of consecutive numbers (powers of two) and consecutive numbers with differences greater than one. Suitable for both tiers.

***Painted Cube***

A cube is painted on the outside then cut into little cubelets by applying the same number of parallel cuts in each dimension. For each possible number of cuts, students count the number of faces painted on each of the resulting cubelets and how many cubelets there are of each type. Students investigate the general problem for a cube, then for a cuboid. Suitable for Foundation.

***Grazing Land***

A horse is tied to a barn (20m by 10m) along one of the long sides. Students investigate how long the rope needs to be in order that the horse may graze a certain area of grass. The task is extended by considering how much grass is available as the horse walks round the corners of the barn. Suitable for both tiers.

| | |
|---|---|
| ***Noughts and Crosses*** | Students investigate the number of winning lines on grids of different sizes. Initially students use a standard three by three noughts and crosses grid which has eight possible winning lines. They then look at other square grids, then rectangular grids. A winning line will have to be defined by each student (e.g. three noughts or crosses in a row, column or diagonal) Suitable for Higher. |

## *AO4 Tasks*

| | |
|---|---|
| ***SHEU*** | Heinemann are currently working with the Schools Health Education Unit. A data set will be available on the Teaching and Learning Software. Visit the SHEU website for more information about the data they collect. The website address is http://www.sheu.org.uk/. Suitable for both tiers. |
| ***Mayfield High School*** | Edexcel has compiled a spreadsheet of data for 1200 fictitious pupils including age, hair and eye colour, height and weight, favourite things, IQ and KS2 results, etc. Students are free to choose their own hypothesis based upon the data but a theme that is commonly used in schools is to compare height and weight by age and gender. The website for this data is http://www.edexcel.org.uk/VirtualContent/97299/006_Mayfield_High_School_Datasheet.xls. Suitable for both tiers. |
| ***Sports Report*** | Students compare how exciting a game of football is in two different countries (e.g. England and Italy). Students can use information such as the average number of goals per game, the percentage attendance and the times at which goals are scored. |

Advised for both tiers. This project is a very good as an introduction to statistical tasks after the Year 9 SAT exams. Students sample say, 30 from 100 matches, then use averages, cumulative frequency and box plot for the percentage attendance and frequency polygons for the times of goals. They do not need stratified sampling or histograms. Suitable for Foundation.

***Read All About It***

This is an AQA set task, but AQA suggest using newspapers or magazines. Book comparisons are also popular; for example, comparing children's literature with adult, or comparing Hemingway and Tolstoy. Suitable for Higher.